



LESTER C. NOECKER SCHOOL
ROSELAND SCHOOL DISTRICT

Math Curriculum

Second Grade

Updated and aligned NJ SLS

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Giuseppe Leone, Superintendent

With special appreciation to Michele Smith & the Math Curriculum Writing Staff

Roseland Mathematics

Grade Level: 2

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Instructional Materials

Everyday Mathematics 4th Edition
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www.everydaymath.com

Supplemental Resources

- Connected Ed <https://connected.mcgraw-hill.com/connected/login.do>
- Illustrative Mathematics <https://www.illustrativemathematics.org/>
- Khan Academy <https://www.khanacademy.org/>
- Math for Elementary School Teachers <http://www.mathforelementaryteachers.org/>
video clips that contain explanations of arithmetic topics including: Place Value/Arithmetic Models/Arithmetic Algorithms, Mental Math, Primes/Divisibility, Fraction Arithmetic, and Word Problems/Model Drawing.
- National Council of Teachers of Mathematics <http://www.nctm.org/>
- National Library of Virtual Manipulatives <http://nlvm.usu.edu/>
- NCTM Illuminations Resources for Teaching Math <http://illuminations.nctm.org/>

Interdisciplinary Connections

Mathematics is a unified body of knowledge whose concepts build upon each other. Connecting mathematical concepts includes linking ideas to related ideas learned previously.

Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas. Students need to connect their mathematical learning to appropriate real-world contexts. They need to create interest and maintain the interest after the novelty of the work has worn off.

Mathematics is the language of science and is greatly utilized in industry and business. It gives us the power to solve difficult real-world problems, but also helps us to understand how the universe operates.

Every mathematics teacher needs to make students unafraid of the subject by convincing the students of the usefulness of learning mathematics in their daily lives and for higher studies. The world today, which leans more and more heavily on Science and Technology, demands more from mathematics. Tomorrow's world will, no doubt, make still greater demands from mathematics.

Interdisciplinary Connections for Grade

Read: *Even Steven Odd Todd* by Kathryn Cristaldi Lesson 1.9 Connection: LA

Read: *Band Aids-Where The Sidewalk Ends* by Shel Silverstein
(Math Masters pg. 170) Lesson 6.5 Connection: LA

Lesson 4.10 Land Of Inch (Supplement) Connection: Social Studies SOC.6.1.4B)

Read: *Picture Pie* by Ed Emberley Lesson 9.1 Connection LA

Read: *Pizza Counting* by Christina Dobson 9.2 Connection LA

Connected Literature:

*If You Were A Fraction

*A Fraction's Goal (Parts Of A Whole)

*The Grouchy Ladybug

*Inch By Inch

New Jersey Student Learning Standards (NJSLS)

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

(4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Understandings	Essential Questions
<p>Students will understand that...</p> <ul style="list-style-type: none"> addition involves adding to and putting together. subtraction involves taking from, taking apart, and comparing. missing numbers in a math sentence can be found using addition and subtraction. a symbol can represent an unknown. the unknown may be located in any position in the equation. objects, drawings, and equations can be used to solve problems. 	<ul style="list-style-type: none"> How can one find the total of parts? How can one find the missing part of a whole?
Knowledge	Skills
<p>Students will know...</p> <ul style="list-style-type: none"> the meaning of addition. the meaning of subtraction. there are multiple interpretations of addition and subtraction. some problems take more than one step to solve. 	<p>Students will be able to . . .</p> <ul style="list-style-type: none"> use addition and subtraction within 100 to solve word problems that involve one-and two-step problems. use objects and drawings to represent problems. use equations with a symbol for the unknown number to represent the problem.
RESOURCES	
<ul style="list-style-type: none"> Everyday Mathematics Lessons: 2-1, 2-2, 2-6, 2-11, 3-6, 3-7, 3-7, 3-8, 4-1, 4-2, 4-4, 4-6, 5-2, 6-2, 6-4, 10-3, 10-4, 10-6 Supplemental Lessons: Binder pages 56-71 	

Operations and Algebraic Thinking

Add and subtract within 20.

2.OA.B.2 With accuracy and efficiency, add and subtract within 20 using mental strategies.

*****By end of Grade 2, know from memory all sums of two one-digit numbers.**

Understandings	Essential Questions
<p>Students will understand that . . .</p> <ul style="list-style-type: none"> there are multiple strategies to add and subtract. 	<ul style="list-style-type: none"> How can a problem be simplified? What strategies are available to determine how much or how many we have?
Knowledge	Skills
<p>Students will know . . .</p> <ul style="list-style-type: none"> numbers that make 10 will help solve problems. numbers can be decomposed into simpler terms. the inverse relationship between addition and subtraction. solutions can be found by forming equivalent but easier or known sums. 	<p>Students will be able to . . .</p> <ul style="list-style-type: none"> fluently add within 20 using mental strategies. fluently subtract within 20 using mental strategies.
RESOURCES	
<ul style="list-style-type: none"> Everyday Mathematics Lessons: 1-4, 1-6, 1-8, 1-11, 2-2, 2-3, 2-4, 2-5, 2-6, 2-7, 2-8, 2-11, 2-12, 2-13, 3-3, 3-4, 4-3, 4-5, 4-7, 5-2, 5-5, 5-7, 6-7, 8-1, 8-2, 8-7, 9-2, 9-3, 9-6, 9-9, 10-7, 12-2, 12-5 Supplemental Lessons: Binder pages 56-67 	

Operations and Algebraic Thinking

Work with equal groups of objects to gain foundations for multiplication.

2.OA.C.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

2.OA.C.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Understandings	Essential Questions
<p>Students will understand that...</p> <ul style="list-style-type: none"> a total number of objects can be found in a rectangular array by finding the sum of equal addends. odd numbers cannot be paired and even numbers can be paired. even numbers can be counted using skip-counting by 2s. 	<ul style="list-style-type: none"> Why would one need to pair things?
Knowledge	Skills
<p>Students will know...</p> <ul style="list-style-type: none"> odd numbers cannot be paired completely and even numbers can. that when counting by 2s, even numbers will finish the group of 	<p>Students will be able to . . .</p> <ul style="list-style-type: none"> determine whether a group of objects (up to 20) has an odd or even number of members use addition to find the total number of objects in a rectangular array (with up to 5 rows and 5 columns). write an equation expressing the total of a rectangular array as a sum of equal addends.
RESOURCES	
<ul style="list-style-type: none"> Everyday Mathematics Lessons: 1-10, 1-12, 2-3, 2-4, 2-5, 2-8, 5-4, 6-6, 6-7, 6-8, 6-9, 8-2, 11-4, 11-6, 11-7 Supplemental Lessons: Binder page 107 	

Number and Operations in Base Ten

Understand place value.

2.NBT.A.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

- a. 100 can be thought of as a bundle of ten tens — called a “hundred.”
- b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.A.2 Count within 1000; skip-count by 5s, 10s, and 100s.

2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

2.NBT.A.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons.

Understandings	Essential Questions
<p>Students will understand that...</p> <ul style="list-style-type: none"> the location of digits in a number determines the value of the number. to compare two numbers, one must compare the digits in each place, starting with the largest place. 	<ul style="list-style-type: none"> Why is place value important?
Knowledge	Skills
<p>Students will know...</p> <ul style="list-style-type: none"> the three digits in a three-digit number represent the amount of hundreds, tens and ones, respectively. the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). 	<p>Students will be able to . . .</p> <ul style="list-style-type: none"> identify one hundred as a bundle of ten tens and ten as a bundle of ten ones. count within 1000. skip-count by 5s, 10s, and 100s. read numbers to 1000. write numbers to 1000 using base-ten numerals, number names, and expanded form. compare three digit numbers using $<$, $=$, and $>$.
RESOURCES	
<ul style="list-style-type: none"> Everyday Mathematics Lessons: 1-1, 1-2, 1-3, 1-5, 1-7, 1-8, 1-9, 1-10, 1-11, 1-12, 2-10, 2-11, 2-12, 3-1, 3-2, 3-4, 3-7, 4-5, 4-9, 6-5, 7-1, 7-7, 10-8, 10-9, 10-10, 10-11, 11-3 Supplemental Lessons: Binder pages 44-45, 115-116 	

Number and Operations in Base Ten

Use place value understanding and properties of operations to add and subtract.

2.NBT.B.5 With accuracy and efficiency, add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.B.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

2.NBT.B.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

2.NBT.B.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

2.NBT.B.9 Explain why addition and subtraction strategies work, using place value and the properties of operations.

Understandings	Essential Questions
<p>Students will understand that . . .</p> <ul style="list-style-type: none"> concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction can help one solve problems. when adding 10 or 100, one must add one to the tens-digit or one to the hundreds-digit and not change the ones-digit. when subtracting 10 or 100, one must subtract one from the tens-digit or one from the hundreds-digit and not change the ones-digit 	<ul style="list-style-type: none"> How does place value help one find the answers to addition and subtraction problems?
Knowledge	Skills
<p>Students will know...</p> <ul style="list-style-type: none"> properties of operations to add and subtract. the values of the digits in a three-digit number. sometimes it is necessary to compose or decompose tens or hundreds. 	<p>Students will be able to . . .</p> <ul style="list-style-type: none"> fluently add and subtract within 100 add up to four two-digit numbers, using strategies using place value and properties of operations. add and subtract within 1000 mentally add 10 or 100 to a given number 100–900. mentally subtract 10 or 100 from a given number 100–900.

	<ul style="list-style-type: none"> • explain why addition and subtraction strategies work, using place value and the properties of operations.
RESOURCES	
<ul style="list-style-type: none"> • Everyday Mathematics Lessons: 1-8, 2-1, 2-2, 2-3, 2-5, 2-6, 2-7, 2-11, 2-12, 2-13, 4-2, 4-6, 4-8, 4-9, 5-1, 6-1, 6-2, 6-4, 6-5, 7-2, 7-3, 9-5, 10-8, 10-9, 10-10, 10-11, 11-1, 11-2, 11-3, 12-2, 12-4 • Supplemental Lessons: Binder pages 56-67, 70-71, 108-110, 115-116 	

Measurement

Measure and estimate lengths in standard units.

2.M.A.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.M.A.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

2.M.A.3 Estimate lengths using units of inches, feet, centimeters, and meters.

2.M.A.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Understandings	Essential Questions
<p>Students will understand that . . .</p> <ul style="list-style-type: none"> the difference between non-standard and standard measurement. measurement tools vary in the size of the unit on them; this variation will affect the choice of tools. 	<ul style="list-style-type: none"> Why do we measure objects? How do we measure objects? Why do we need standard units of measurement?
Knowledge	Skills
<p>Students will know...</p> <ul style="list-style-type: none"> appropriate tools must be used in order to properly measure an object. the approximate length of an inch, foot, centimeter, and meter. 	<p>Students will be able to . . .</p> <ul style="list-style-type: none"> select an appropriate tool to measure an object. measure the length of an object. measure the length of an object with two different tools. describe how the measurements of one object differ when using two different tools (relate the measurement to the size of the unit chosen). estimate lengths using units of inches, feet, centimeters, and meters. measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

RESOURCES

- Everyday Mathematics Lessons:** 4-7, 7-6, 7-7, 7-8, 9-1, 9-2, 9-3, 9-4
- Supplemental Lessons:** Binder pages 72-106, 111-114

Measurement

Relate addition and subtraction to length.

2.M.B.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

2.M.B.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Understandings	Essential Questions
<p>Students will understand that . . .</p> <ul style="list-style-type: none"> • addition and subtraction can be used to solve word problems involving lengths that are given in the same units. • whole numbers can be represented as the lengths from 0 to the number located on an equally-spaced number line. • whole-number sums and differences can be represented on a number line. 	<ul style="list-style-type: none"> • How are the locations of numbers on a number line related to length? • How can addition and subtraction be used to find lengths?
Knowledge	Skills
<p>Students will know...</p> <ul style="list-style-type: none"> • drawings (such as drawings of rulers) can be used to solve problems involving length. • equations with an unknown can be used to solve problems involving length 	<p>Students will be able to . . .</p> <ul style="list-style-type: none"> • add within 100 to solve word problems involving length. • subtract within 100 to solve word problems involving length. • represent whole numbers on a number line as length from 0. • represent whole numbers sums and differences with 100 on a number-line diagram
RESOURCES	
<ul style="list-style-type: none"> • Everyday Mathematics Lessons: 1-1, 1-8, 2-1, 2-4, 2-6, 2-10, 2-12, 3-6, 4-2, 4-4, 4-6, 5-3, 6-1, 6-2, 6-3, 6-5, 6-7, 7-3, 7-7, 7-8, 8-1, 11-1, 11-2, 12-3 • Supplemental Lessons: Binder pages 44-45, 70-71, 115-116 	

Measurement

Work with time and money.

2.M.C.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

2.M.C.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?

Understandings	Essential Questions
<p>Students will understand that . . .</p> <ul style="list-style-type: none"> when time passes, the hour hand and the minute hand move at different rates. different coins have different values, not related to the size of the coin. 	<ul style="list-style-type: none"> How do the positions of the hands on an analog clock indicate the time? How do the numbers on a digital clock indicate the time? How do we determine how much money is needed and how money one has?
Knowledge	Skills
<p>Students will know...</p> <ul style="list-style-type: none"> between the hour hand and the minute hand. on an analog clock, on the hour, the hour hand is pointing exactly to the number that represents the hour; on the half-hour, the hour hand is pointing exactly half-way between two numbers. on a digital clock, the digits to the left of the colon represent the hour and the digits to the right of the colon represent the minutes. the value of a dollar bill, quarter, dime, nickel and penny. 	<p>Students will be able to . . .</p> <ul style="list-style-type: none"> tell and write time to the nearest five minutes using a.m. and p.m., on an analog clock. tell and write time to the nearest five minutes using a.m. and p.m., on a digital clock. solve word problems involving dollar bills, quarters, dimes, nickels and pennies using \$ and ¢ symbols appropriately.
RESOURCES	
<ul style="list-style-type: none"> Everyday Mathematics Lessons: 1-3, 1-5, 1-6, 1-9, 3-2, 3-3, 3-7, 3-8, 4-2, 4-3, 4-5, 4-6, 5-1, 6-6, 7-5, 10-1, 10-3, 10-4, 10-5, 10-6, 11-1, 11-2, 12-1, 12-2 Supplemental Lessons: Binder pages 70-71, 119-133 	

Data and Literacy

Understand concepts of data.	
2.DL.A.1 Understand that people collect data to answer questions. Understand that data can vary.	
2.DL.A.2 Identify what could count as data (e.g., visuals, sounds, numbers).	
Understandings	Essential Questions
Students will understand that... <ul style="list-style-type: none"> • Use data to answer questions • There can be different types of data 	<ul style="list-style-type: none"> • How can we use data to answer questions?
Knowledge	Skills
Students will know... <ul style="list-style-type: none"> • What is data? • How to collect data? • How to understand data? 	Students will be able to . . . <ul style="list-style-type: none"> • collect different types of data • chart data • talk about data and analyze it
RESOURCES	
<ul style="list-style-type: none"> • Everyday Mathematics 4 Lessons: 6-1, 6-4, 7-3, 7-8, 7-9, • Supplemental Lessons: Favorite Lunch (Sept.), Favorite Halloween Candy (Oct.), Favorite Thanksgiving Food (Nov.), Favorite Holiday Activity(Dec.), Favorite Crayon Color/Opinion Writing/The Day the Crayons Quit (Jan.), Groundhog Day/Groundhog See Shadow (Feb.), Track Weather for a Week (Apr.),Favorite Summer Activity (May) 	

Data and Literacy	
Represent and interpret data.	
2.DL.B.3 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	
2.DL.B.4 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.	
Understandings	Essential Questions
Students will understand that . . . <ul style="list-style-type: none"> • There are many ways to analyze data. 	<ul style="list-style-type: none"> • How can representing data help us to interpret it and draw conclusions?
Knowledge	Skills
Students will know...	Students will be able to . . .

<ul style="list-style-type: none"> the difference between a picture graph and a bar graph. how to make a line plot. 	<ul style="list-style-type: none"> generate measurement data by measuring lengths of several objects to the nearest whole unit generate measurement data by making repeated measurements of the same object. show measurements by making a line plot, where the horizontal scale is marked off in whole-number units. organize data with up to four categories. represent data with up to four categories using a picture graph. represent data with up to four categories using a bar graph. solve simple put-together, take-apart, and compare problems using a bar graph
RESOURCES	
<ul style="list-style-type: none"> Everyday Mathematics Lessons: 3-5, 6-3, 6-6, 7-6, 7-8, 9-1, 9-4, 9-8, 10-5, 11-3, 12-6, 12-7 Supplemental Lessons: Binder pages 117-118,134-155 	

Geometry

Reason with shapes and their attributes.

2.G.A.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.¹ Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

2.G.A.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

2.G.A.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. *For example, students partition a rectangle (i.e., the whole) into three equal shares, identify each of the shares as a 'third' and describe the rectangle as three 'thirds'.*

Understandings	Essential Questions
<p>Students will understand that . . .</p> <ul style="list-style-type: none"> • shares of a whole must always be equal. • decomposing into more equal shares creates smaller shares. • equal shares of identical wholes need not have the same shape. 	<ul style="list-style-type: none"> • Why do we need to identify shapes? • Why would we partition shapes?
Knowledge	Skills
<p>Students will know...</p> <ul style="list-style-type: none"> • the characteristics of triangles, quadrilaterals, pentagons, hexagons, and cubes. • the word half, third, and fourth refers, respectively, to having 2, 3, and 4 equal parts. 	<p>Students will be able to . . .</p> <ul style="list-style-type: none"> • recognize shapes having specified attributes. • draw shapes having specified attributes. • identify triangles, quadrilaterals, pentagons, hexagons, and cubes. • partition a rectangle into rows and columns of the same-size squares. • count the squares in a partitioned rectangle to find the total number. • partition circles into two, three, or four equal shares. • partition rectangles into two, three, or four equal shares. • appropriately use the words <i>halves</i>, <i>thirds</i>, <i>fourths</i> and <i>quarters</i> and the phrases <i>half of</i>, <i>a third of</i>, <i>a fourth of</i>, and <i>quarter of</i>. • describe the whole as two halves, three thirds, or four fourths. • identify equal shares of identical wholes even though they do not have the same shape.

RESOURCES
<ul style="list-style-type: none">• Everyday Mathematics Lessons: 3-4, 4-3, 4-7, 5-1, 5-3, 5-4, 5-5, 5-6, 5-7, 8-1, 8-2, 8-3, 8-4, 8-5, 8-6, 8-7, 9-7, 10-3, 10-7• Supplemental Lessons: Binder page 107

Connecting the Standards for Mathematical Content to the Standards for Mathematical Practice

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards, which set an expectation of understanding, are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a

problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Standard 9 21st Century Life and Careers

In today's global economy, students need to be lifelong learners who have the knowledge and skills to adapt to an evolving workplace and world. To address these demands, Standard 9, 21st Century Life and Careers, which includes the 12 Career Ready Practices, establishes clear guidelines for what students need to know and be able to do in order to be successful in their future careers and to achieve financial independence.

Mission: *21st century life and career skills enable students to make informed decisions that prepare them to engage as active citizens in a dynamic global society and to successfully meet the challenges and opportunities of the 21st century global workplace.*

Vision: To integrate 21st Century life and career skills across the K-12 curriculum and in Career and Technical Education (CTE) programs to foster a population that:

- Continually self-reflects and seeks to improve the essential life and career practices that lead to success.
- Uses effective communication and collaboration skills and resources to interact with a global society.
- Is financially literate and financially responsible at home and in the broader community.
- Is knowledgeable about careers and can plan, execute, and alter career goals in response to changing societal and economic conditions.
- Seeks to attain skill and content mastery to achieve success in a chosen career path.

The Standards: Standard 9 is composed of the Career Ready Practices and Standard 9.1, 9.2, and 9.3 which are outlined below:

- **The 12 Career Ready Practices**
These practices outline the skills that all individuals need to have to truly be adaptable, reflective, and proactive in life and careers. These are researched practices that are essential to career readiness.
- **9.1 Personal Financial Literacy**
This standard outlines the important fiscal knowledge, habits, and skills that must be mastered in order for students to make informed decisions about personal finance. Financial literacy is an integral component of a student's college and career readiness, enabling students to achieve fulfilling, financially-secure, and successful careers.
- **9.2 Career Awareness, Exploration, and Preparation**
This standard outlines the importance of being knowledgeable about one's interests and talents, and being well informed about postsecondary and career options, career planning, and career requirements.
- **9.3 Career and Technical Education**
This standard outlines what students should know and be able to do upon completion of a CTE Program of Study.

For students to be college and career ready they must have opportunities to understand career concepts and financial literacy. This includes helping students make informed decisions about their future personal, educational, work, and financial goals. By integrating Standard 9 into instruction, New Jersey students will acquire the necessary academic and life skills to not only achieve individual success but also to contribute to the success of our society.

21st Century Themes

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

CRP1. Act as a responsible and contributing citizen and employee.

CRP2. Apply appropriate academic and technical skills.

CRP3. Attend to personal health and financial well-being.

CRP4. Communicate clearly and effectively and with reason.

CRP5. Consider the environmental, social and economic impacts of decisions.

CRP6. Demonstrate creativity and innovation.

CRP7. Employ valid and reliable research strategies.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP9. Model integrity, ethical leadership and effective management.

CRP10. Plan education and career paths aligned to personal goals.

CRP11. Use technology to enhance productivity.

CRP12. Work productively in teams while using cultural global competence.

CRP1. Act as a responsible and contributing citizen and employee

Career-ready individuals understand the obligations and responsibilities of being a member of a community, and they demonstrate this understanding every day through their interactions with others. They are conscientious of the impacts of their decisions on others and the environment around them. They think about the near-term and long-term consequences of their actions and seek to act in ways that contribute to the betterment of their teams, families, community and workplace. They are reliable and consistent in going beyond the minimum expectation and in participating in activities that serve the greater good.

CRP2. Apply appropriate academic and technical skills.

Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation

CRP3. Attend to personal health and financial well-being.

Career-ready individuals understand the relationship between personal health, workplace performance and personal well-being; they act on that understanding to regularly practice healthy diet, exercise and mental health activities. Career-ready individuals also take regular action to contribute to their personal financial wellbeing, understanding that personal financial security provides the peace of mind required to contribute more fully to their own career success.

CRP4. Communicate clearly and effectively and with reason.

Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others' time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

CRP5. Consider the environmental, social and economic impacts of decisions.

Career-ready individuals understand the interrelated nature of their actions and regularly make decisions that positively impact and/or mitigate negative impact on other people, organization, and the environment. They are aware of and utilize new technologies, understandings, procedures, materials, and regulations affecting the nature of their work as it relates to the impact on the social condition, the environment and the profitability of the organization.

CRP6. Demonstrate creativity and innovation.

Career-ready individuals regularly think of ideas that solve problems in new and different ways, and they contribute those ideas in a useful and productive manner to improve their organization. They can consider unconventional ideas and suggestions as solutions to issues, tasks or problems, and they discern which ideas and suggestions will add greatest value. They seek new methods, practices, and ideas from a variety of sources and seek to apply those ideas to their own workplace. They take action on their ideas and understand how to bring innovation to an organization.

CRP7. Employ valid and reliable research strategies.

Career-ready individuals are discerning in accepting and using new information to make decisions, change practices or inform strategies. They use reliable research process to search for new information. They evaluate the validity of sources when considering the use and adoption of external information or practices in their workplace situation.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

CRP9. Model integrity, ethical leadership and effective management.

Career-ready individuals consistently act in ways that align personal and community-held ideals and principles while employing strategies to positively influence others in the workplace. They have a clear understanding of integrity and act on this understanding in every decision. They use a variety of means to positively impact the directions and actions of a team or organization, and they apply insights into human behavior to change others' action, attitudes and/or beliefs. They recognize the near-term and long-term effects that management's actions and attitudes can have on productivity, morals and organizational culture.

CRP10. Plan education and career paths aligned to personal goals.

Career-ready individuals take personal ownership of their own education and career goals, and they regularly act on a plan to attain these goals. They understand their own career interests, preferences, goals, and requirements. They have perspective regarding the pathways available to them and the time, effort, experience and other requirements to pursue each, including a path of entrepreneurship. They recognize the value of each step in the education and experiential process, and they recognize that nearly all career paths require ongoing education and experience. They seek counselors, mentors, and other experts to assist in the planning and execution of career and personal goals.

CRP11. Use technology to enhance productivity.

Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.

CRP12. Work productively in teams while using cultural global competence.

Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.

Differentiation Strategies

Students with Disabilities/ Students at Risk of School Failure

(For students with disabilities, appropriate accommodations, instructional adaptations, and/or modifications should be determined by the IEP or 504 team)

Modifications for Classroom

- Pair visual prompts with verbal presentations
- Ask students to restate information, directions, and assignments.
- Give repetition and practice exercises
- Model skills/techniques to be mastered
- Give extended time to complete class work
- Provide copy of class notes
- Determine if preferential seating would be beneficial
- Provide access to a computer
- Provide copies of textbooks for home
- Provide access to books on tape/CD/digital media, as available and appropriate
- Assign a peer helper in the class setting
- Provide oral reminders and check student work during independent work time
- Assist student with long and short term planning of assignments
- Encourage student to proofread assignments and tests
- Provide regular parent/school communication

Modifications for Homework and Assignments

- Provide extended time to complete assignments
- Break down assignments
- Provide the student with clearly stated (written) expectations and grading criteria for assignments
- Implement RAFT activities as they pertain to the types/modes of communication (role, audience, format, topic)

Modifications for Assessments

- Provide extended time on classroom tests and quizzes
- Provide alternate setting as needed
- Restate, reread, and clarify directions/questions
- Distribute study guide for classroom tests
- Establish procedures for accommodations /modifications for assessments

Differentiation Strategies
Gifted and Talented
<p><i>(content, process, product and learning environment)</i></p> <ul style="list-style-type: none"> • Allow students to pursue independent projects based on their individual interests • Provide enrichment activities that include more advanced material • Allow team-teaching opportunities and collaboration • Set individual goals • Conduct research and provide presentation of appropriate topics • Design surveys to generate and analyze data to be used in discussion. • Use Higher-Level Questioning Techniques • Provide assessments at a higher level of thinking
English Language Learners
<p>Modifications for Classroom</p> <ul style="list-style-type: none"> • Pair visual prompts with verbal presentations • Provide repetition and practice • Model skills/techniques to be mastered <p>Modifications for Homework/Assignments</p> <ul style="list-style-type: none"> • Provide Native Language Translation (peer, online assistive technology, translation device, bilingual dictionary) • Provide extended time for assignment completion as needed • Highlight key vocabulary • Use graphic organizers